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THRESHOLDS FOR SEVERAL ANALOG MODULATION SYSTEMS

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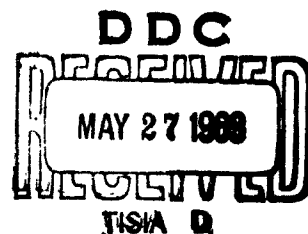
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## INTRODUCTION AND SUMMARY

The onset of a threshold in communication systems is a familiar phenomenon. By threshold is meant an abrupt degradation in performance when the signal-to-noise ratio at the input to the detector falls below some more or less critical level. Two common examples are:

- (1) the conventional FM threshold when a discriminator is the detector for angle-modulated signals, and
- (2) the small signal suppression effect when power law detectors are used for AM signals.

This report is intended as a compilation of selected works of others on the thresholds of several analog modulation schemes operating in additive white, gaussian noise. One threshold curve, ( $S^*$ ), derived from information theoretic considerations, may be interpreted as representing the best possible performance of a broad class of modulation-demodulation schemes. The remaining threshold curves, ( $E, D, V, R$ ), refer to angle modulation and various demodulators.

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\* Refer to Fig. 1.

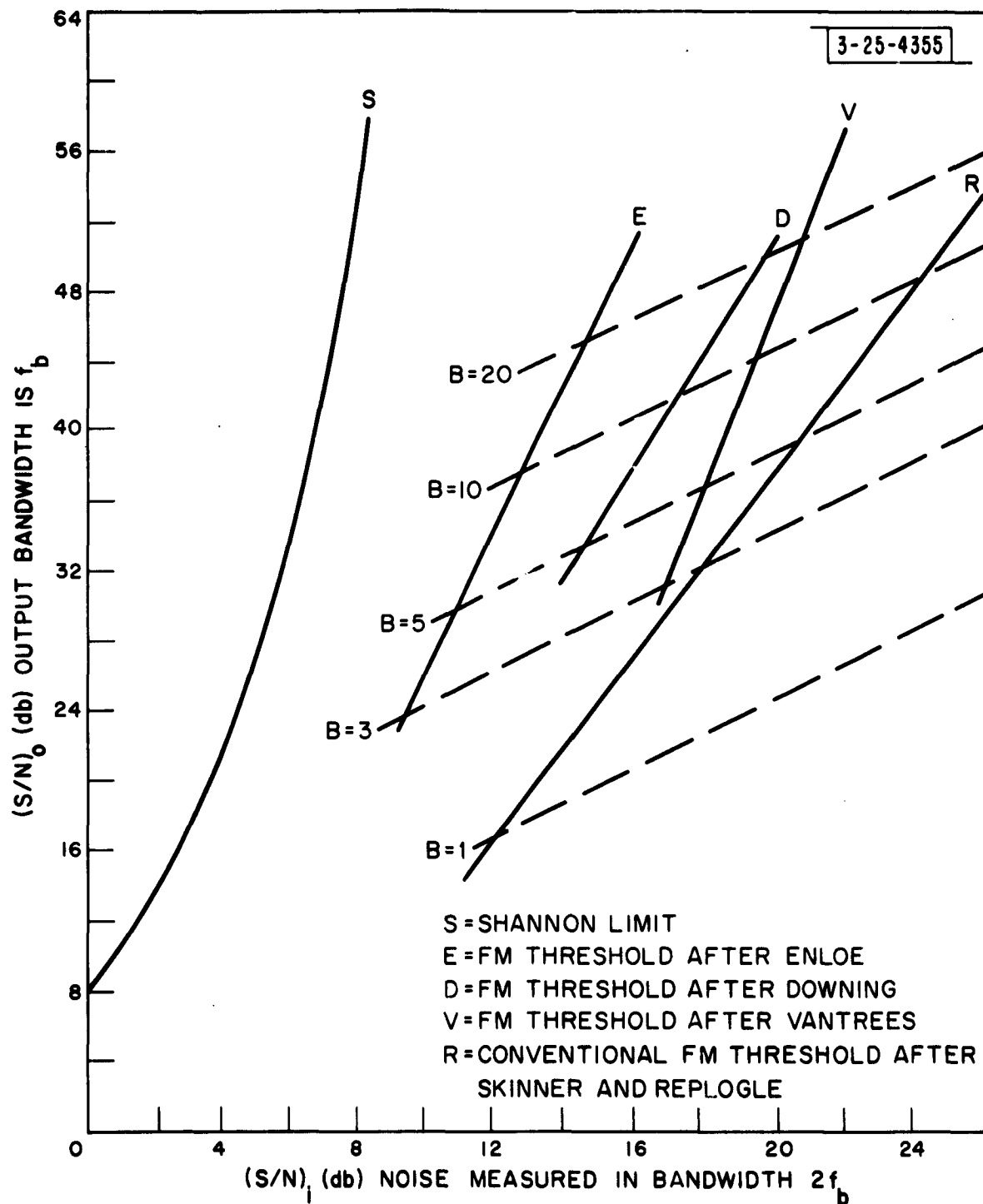


Fig. 1 Threshold Curves

Figure 1 is a curve comparing thresholds of several analog modulation and detection systems. With reference to Fig. 2, all curves are derived on the basis of the following assumptions:

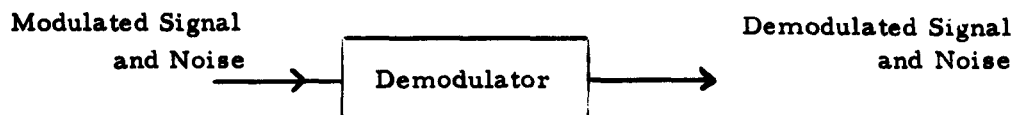


Fig. 2 System Model

1. An analog signal confined to bandwidth  $f_b$  is modulated onto an RF carrier.
2. The modulated carrier contains average power  $S$  watts.
3. The modulated carrier may occupy a bandwidth arbitrarily large.
4. The noise input to the demodulator is white and gaussian, with single-sided spectral density  $N_o \frac{\text{watts}}{\text{cps}}$ .
5. The input signal-to-noise ratio is defined as

$$\left(\frac{S}{N}\right)_i = \frac{S}{2f_b N_o}$$

6. The demodulated signal and output noise are both confined to a low-pass single-sided bandwidth  $f_b$ .

The curves of Fig. 1 are now briefly discussed.

I. The output of a data source delivering information at the rate  $R$  nats per second is coded into a gaussian-like signal of bandwidth  $W$ . If an optimum decoder is used, arbitrarily low error can result if

$$R < C_{in} = W \ln \left( 1 + \frac{S}{N_o W} \right) \leq \frac{S}{N_o} \frac{\text{nats}}{\text{sec}} \quad (1)$$

The second inequality in Eq. (1) follows from

$$\ln(1+x) \leq x, \quad x > -1$$

If  $W$  is permitted to become arbitrarily large, Eq. (1) becomes

$$R < C_{in} = \frac{S}{N_o} \frac{\text{nats}}{\text{sec}} \quad (2)$$

Suppose the action of the demodulator is to "compress" input signal into an output confined to bandwidth  $f_b$ . This compression may involve arbitrarily long delays. Suppose the noise at the demodulator output is additive and gaussian, and the signal is average power limited. The output of the demodulator is to be decoded. The maximum rate at which this decoder may operate with arbitrarily low error is

$$C_{out} = f_b \ln \left[ 1 + \left( \frac{S}{N_o} \right) \right] \quad (3)$$

For decoding to be possible,  $R$  must be less than  $C_{out}$ .

The action of the demodulator is that of signal processing. It cannot increase information; at best, it will not destroy information. Thus

$$C_{out} \leq C_{in} \quad (4)$$

Substituting Eqs. (2) and (3) into Eq. (4), we obtain

$$\frac{S}{N_o} \geq f_b \ln \left[ 1 + \left( \frac{S}{N} \right)_o \right]$$

or

$$\left( \frac{S}{N} \right)_{in} \geq 1/2 \ln \left[ 1 + \left( \frac{S}{N} \right)_o \right] \quad (5)$$

Eq. (5), with the equal sign, is plotted as Curve S. It may be viewed as the threshold limit of any demodulator whose input is a wideband-modulated signal in gaussian noise, and whose output is a bandlimited signal in gaussian noise.\* Practical demodulators will have operating points to the right of Curve S.

II. Curve R is a plot of conventional FM modulation using a discriminator as a detector. It is due to Skinner and Replogle and is quoted in Ref. [2]. Overlaid on Curve R is a family of lines of slope 1 identified by the parameter  $\beta$ . This is the deviation ratio of the FM. To the right of Curve R, the  $\left( \frac{S}{N} \right)_o$  vs  $\left( \frac{S}{N} \right)_i$  characteristic has unity slope and follows a line of corresponding  $\beta$ . To the left of Curve R,  $\left( \frac{S}{N} \right)_o$  changes markedly for small change in  $\left( \frac{S}{N} \right)_i$ .

III. Enloe<sup>[2,3]</sup> has recently analyzed an old proposal by Chaffee<sup>[4]</sup> to perform FM detection in a feedback loop. Enloe predicts a lowering of the FM threshold as shown by Curve E.\*\*

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\*Equation (5) is derived from a similar point of view by J. A. Develet, (See Ref. [1]).

\*\*This curve does not account for phase shift in the loop. In Ref. [2], Enloe shows how the threshold is increased for excess phase shift at band edge.

IV. Downing<sup>[5]</sup> has analyzed Enloe's model, and making some more conservative assumptions, arrives at the more conservative threshold of Curve D.

V. VanTrees<sup>[6]</sup> has analyzed an FM or PM detector using a phase-locked loop. This contrasts with the work of Enloe and Downing who assume a discriminator operating at bandpass in the loop. In VanTrees' analysis, the threshold may be defined as corresponding to a given RMS phase error at the output of the phase detector. This phase error is attributable to both input noise and dynamic effects (i.e., phase shift at the modulating frequency). The threshold corresponding to an RMS phase error of  $\frac{\pi}{8}$  radians is shown in Curve V. A larger allowable RMS phase error would move the threshold curve along a line of constant  $\beta$  by an amount equal to  $25 \log_{10} \frac{\text{allowable RMS phase error}}{\frac{\pi}{8}}$  db.\*

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\*Increasing the allowable RMS phase error decreases the threshold.

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